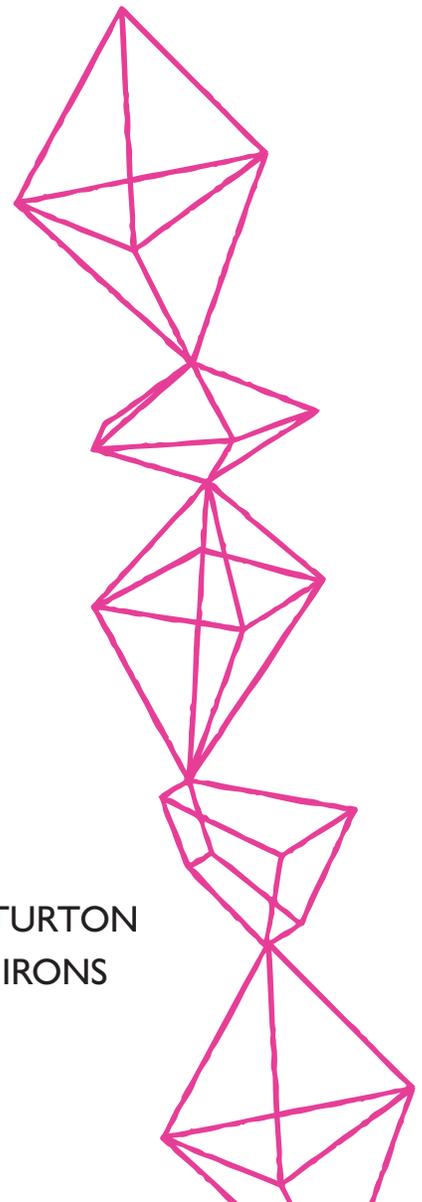


# **geo** **faces and frames**

Exploring Properties of 3D Shapes

Sample Activities



ALLAN TURTON  
CALVIN IRONS



### Materials

- Clay, fishing line, and cutting board — for each pair of students
- Large sheet of paper, approx. 30 cm × 40 cm — 1 for each pair of students

### Did You Know?

Parallel lines are the same distance apart over their entire lengths. Parallel planes are the same distance apart over their entire surfaces.

### Did You Know?

Shapes that are exactly the same size and shape are called congruent shapes. Shapes that look the same, except for their size, are called similar shapes.

The shape that remains after the top of a pyramid or cone is sliced off parallel to the base is called a frustum.

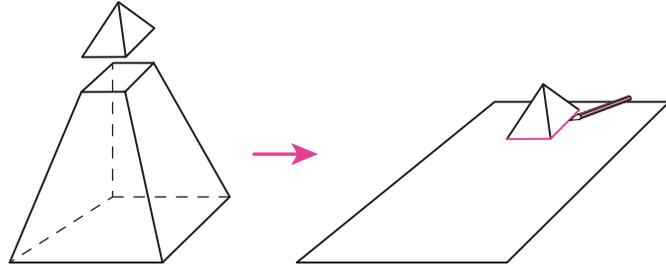
## 3. Cross-Sections

### Preparation

No preparation is required.

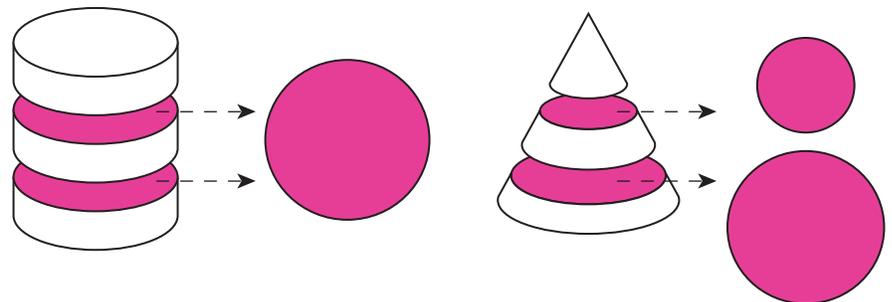
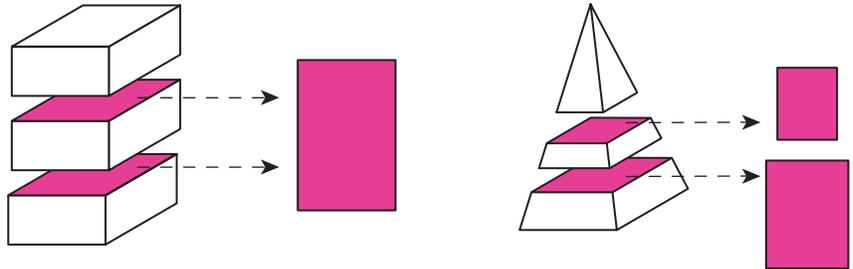
### Activity

1. Direct each pair of students to make a prism and a pyramid, or a cone and a cylinder. Review what the students know about the term “parallel” and instruct each pair to make several cuts parallel to the base of each shape, starting at the top and gradually working down.



▲ After each cut is made, have the students trace the plane section to record the result.

2. As they cut each section, have the students trace the plane section, starting at the top of the sheet of paper. Ask, *What do you notice about the shape of the plane sections of the prisms and cylinders?* (They are the same shape and size.) *What about the pyramids and cones?* (The plane sections are the same shape but different sizes.) Bring out that these observations apply to all prisms, pyramids, cones, and cylinders, and that plane sections made from cuts parallel to the base of these shapes are called “cross-sections”.



▲ Prisms and cylinders have uniform cross-sections, but pyramids and cones do not.

Examining certain plane sections can reveal whether a shape has reflective or rotational symmetry. See *Simple Symmetry* for more information and activities on symmetry.

## 4. Cuboctahedron

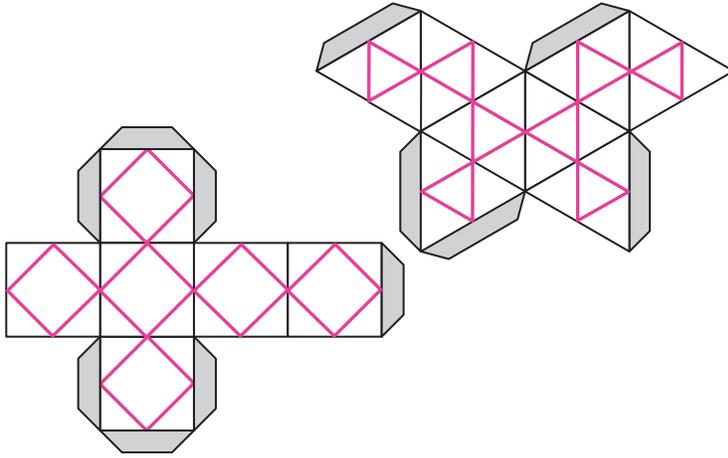


### Preparation

Make one copy of Blackline Master 12 for half the number of student groups. Cut the copies in half so that you have equal numbers of the hexahedron and octahedron nets.

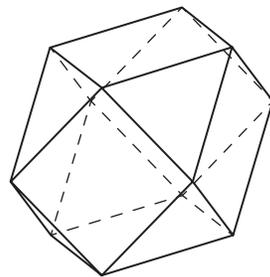
### Activity

1. Provide each group with one net and have the students cut out their net. Ask one student in each group to locate the midpoint on the edges of each face and then draw lines to join the midpoints using a color pencil.



▲ Instruct the students to connect the midpoints on the edges of each face.

2. Have another student construct the net. Say, *Look at where the lines that you drew connect with each other. If you slice along these lines, what shapes will the new faces be?* (Triangles or squares.)
3. Challenge each group to use the Polydron pieces to make the three-dimensional shapes that would result if the existing shapes were cut along the drawn lines. Once they have completed the task, explain that “truncating” a shape means cutting some or all of its corners off to produce a “truncated” shape. Ask the students to suggest names for the shapes they have made. Some students may suggest “truncated cube”, while others may suggest “truncated octahedron”. Both suggestions are correct because the students will have made identical shapes by using the same method of truncating the cube and the octahedron. This resulting shape is given the name “cuboctahedron”.



▲ Truncating the cube and regular octahedron in the same way results in the same shape.

### Materials

- Blackline Master 12 (page 70)
- Scissors, glue, and ruler — for each group of students
- Color pencil — 1 for each group of students
- Polydron shapes or similar — 8 triangles and 6 squares for each group of students

### Did You Know?

The cuboctahedron is one of 13 special truncated shapes. Known as the Archimedean solids, these shapes have regular faces of at least two different types that are arranged so that the number and type of faces that meet at each vertex are the same. All the edges are the same length. The shapes are named after the Greek mathematician Archimedes (3rd century BC).

### Did You Know?

Truncating an icosahedron produces a 32-faced shape that has 12 pentagons, each surrounded by 5 hexagons. The outer shells of some soccer balls have the same pattern.



## 2. Identifying Bases

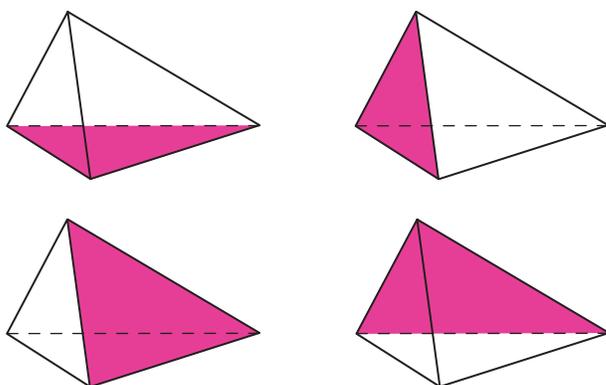


### Preparation

No preparation is required.

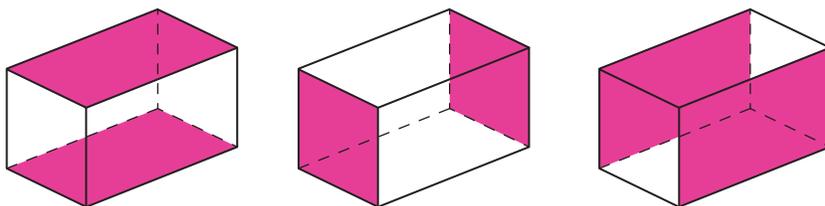
### Activity

1. Complete this activity with one small group of students at a time. Select a square-based pyramid from the set of shapes and ask, *Which face of this pyramid is not like the others? Why?* Invite a volunteer to identify the square face as different because the other faces are triangles. Say, *We call this face the base of the pyramid. The pyramid usually rests on this face to help us examine it, but it can rest on any of the other faces.*
2. Direct the students to identify the bases of other pyramids in the set. Some students may have difficulty identifying the base of a triangular-based pyramid. Explain that with these types of pyramids, any face can be called the base.



▲ Because triangular-based pyramids have all triangular faces, any face can be the base.

3. Say, *Prisms have two bases. The bases are identical and parallel, and they are joined together by rectangles (including squares).* If the students are unfamiliar with the term “parallel face”, explain that parallel faces are the same distance apart across their entire surfaces.
4. Display a triangular-based prism. Invite a volunteer to identify two identical faces. Depending on the type of triangular-based prism, the student may identify not only the triangular faces, but also two of the other faces. If this happens, reinforce that the bases must be parallel and joined by rectangles.
5. Encourage the students to identify the bases of other prisms. When the students examine a rectangular-based prism (including a cube), explain that any pair of parallel faces can be called the bases.



▲ Because rectangular-based prisms have all rectangular faces, any pair of parallel faces can be called the bases.

### Materials

- Set of right prisms (including cubes) and pyramids

### Did You Know?

Prisms do not always have only rectangular faces joining the bases: the faces can be any parallelogram. See Activity 7 (“Oblique Prisms”) on page 44 for more details.

A rectangular-based prism is also known as a cuboid.